Propagation of TE Modes in Dielectric Loaded Waveguides

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Abstract—The propagation of TE_{no} modes in rectangular waveguides that contain two dielectric slabs parallel to the narrow wall and extending over the full height of the guide is investigated. Waveguide and dielectric are assumed to be lossless and infinitely long. Apart from these restrictions, the dielectric slabs may have arbitrary thickness, position, and dielectric constant. The analysis is restricted to TE_{no} modes with the E-field parallel to the narrow guidewall. The guide containing only one dielectric slab is covered by this analysis. The even modes n=2, 4, 6, $\cdots$ of the guide with two slabs correspond to the odd modes $n'=n/2=1, 2, 3, \cdots$ of the guide with one slab half the width of the guide with two slabs.

For six relative dielectric constants ($\epsilon=2.25$, 4.00, 9.00, 12.25, 16.00, 25.00) the cutoff frequencies for TE 10, 20, 30, 40, 60 modes and the normalized propagation constants for TE 10 and TE 20 modes between their respective cutoff frequencies and a frequency slightly above the second- and fourth-order mode cutoff frequency for the empty guide, respectively, have been computed for a large range of slab thicknesses and slab positions. Selected results are presented graphically.

These results are discussed. The parametric dependences of field distributions, of normalized characteristic impedances, of the ratio of cutoff frequencies (fractional bandwidth), and of the ratio of magnetic field components (ellipticity) are illustrated.

List of Symbols

h = waveguide height (meters)

$2w$ = waveguide width (meters)

$\alpha=a/w$, $\delta=d/w$, $\gamma=c/w$ = waveguide dimensions as shown in Fig. 1 with $\alpha+\delta+\gamma=1$

i_x, i_y, i_z = unit vectors

x, y, z = right-hand coordinate system as shown in Fig. 1 (meters)

$\phi=x/w$ = normalized x-coordinate

$B=(2\pi/\lambda_0)w$ = free-space propagation number (frequency parameter)

$B_c=(2\pi/\lambda_c)w$ = normalized cutoff frequency

$K=(2\pi/\lambda_g)w$ = longitudinal propagation number in the guide

$P=(2\pi/\lambda_e)w$ = transverse propagation number in empty part of guide

$Q=(2\pi/\lambda_d)w$ = transverse propagation number in the dielectric

$\rho_1, \cdots \rho_6$ = electrical widths of waveguide sections (radians)

θ = phase angle (radians)

A, C, D = relative amplitudes

μ_0 = free-space permeability ($V\cdot s\cdot A^{-1}\cdot m^{-1}$)

ϵ_0 = free-space dielectric constant ($A\cdot s\cdot V^{-1}\cdot m^{-1}$)

ϵ = relative dielectric constant of dielectric

$\vec{E}=i_zE_z$ = electric field ($V\cdot m^{-1}$)

E_0 = normalizing electric field ($V\cdot m^{-1}$)

$\vec{H}=i_xH_x+i_yH_y$ = magnetic field ($A\cdot m^{-1}$)

H_0 = normalizing magnetic field ($A\cdot m^{-1}$)

Z_0 = wave impedance of free space ($V\cdot A^{-1}$)

Z_w = wave impedance of guide ($V\cdot A^{-1}$)

Z_i, Z_v, Z_r = characteristic impedances of guide ($V\cdot A^{-1}$)

z_i, z_v = normalized characteristic impedances

Ω = fractional bandwidth

ELL = ellipticity

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Introduction

THE EXACT SOLUTION of propagation problems in waveguides containing dielectric slabs often serves as a basis for perturbation calculations for the same problem in waveguides containing ferrite [13]. It is for this reason that some of the previous analyses have been performed [7], [14] and that the one presented here has been undertaken. Applications of these results are shown elsewhere [11], [15], [17].

Previous analyses of propagation of TE_{no} modes in rectangular waveguides which contain dielectric slabs have, in general, been restricted to two cases i.e., where the dielectric slab is placed a) against a waveguide wall [2], [3], [5], [7], [9], [10], or b) in the center plane of the waveguide [3], [5], [7], [8], [11]. A few more special cases have been considered by investigators dealing with ferrite applications in the microwave region [7], [13]. For a rather general position of the dielectric slab, expressions to obtain the propagation constant have been given [6], [9] and certain phase-shift characteristics of the loaded guide have been calculated [12].

The present analysis deals with a rectangular waveguide that contains two dielectric slabs parallel to the narrow walls and extending over the full height of the guide. The slabs are placed symmetrically with respect to the center E-plane of the guide. Apart from this restriction, the slabs have arbitrary position, thickness, and dielectric constant. Only TE_{no} modes with E-fields parallel to the narrow guidewall are considered. The guide and the dielectric are assumed to be lossless and infinitely long. Waveguides containing only one dielectric slab are covered by this analysis.

A compact but complete theoretical treatment of the problem is given. The parametric dependence of the primary computer results, that is, the normalized cutoff frequencies and propagation constants, and of secondary results, such as impedances, field distributions, and fractional bandwidths, is illustrated.

Theory

Figure 1 shows the cross section of a rectangular, dielectric loaded waveguide in a rectangular coordinate system. The broad dimension of the guide extends along the x-axis; y is the direction of propagation of fields in the guide; the height h of the guide extends in the z-direction; a, c, d, w are waveguide dimensions along the x-axis. $\beta=\omega_0\sqrt{\mu_0\epsilon_0}$ is the free-space propagation constant; k is the propagation constant in the guide in the direction of the guide; p in the empty region and q in the dielectric are propagation constants transverse to the direction of the guide and the electric field. Instead of these symbols, dimensionless quantities will be used throughout the analysis. These are obtained by either

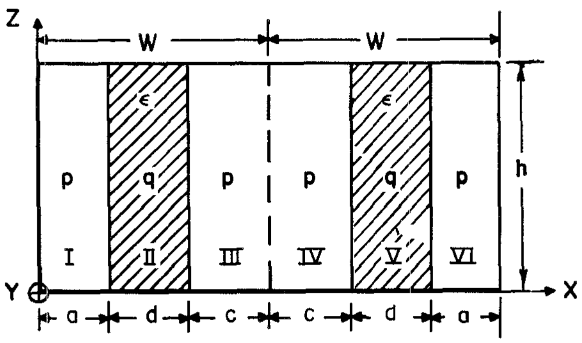

Fig. 1. Waveguide with dielectric slabs.

multiplying or dividing the quantities given above by w, e.g., $B=\beta w$, $K=kw$, $P=pw$, $Q=qw$, and $\phi=x/w$, $\alpha=a/w$, $\gamma=c/w$, $\delta=d/w$, where $\alpha+\gamma+\delta=1$.

Field Distribution

It suffices, because of the symmetry of the loaded guide, to consider the regions I, II, III of Fig. 1, so that $0\leq x\leq w$ or $0\leq\phi\leq 1$. All fields vary as $\exp j(\omega t-ky)$, so that $\partial/\partial t=j\omega$ and $\partial/\partial y=-jk$. This t and y dependence is omitted in all equations. The relative permeability of the dielectric is assumed to be unity.

The E-fields in the various regions of the guide can tentatively be described by the dimensionless shape function $g(\phi)=E_z(\phi)/E_0$ given in Table I. E_o is a normalizing field strength. The compatibility of these assumptions with Maxwell's equations has to be shown. These equations for the problem considered here reduce to

$$\nabla\times\vec{E}=i_x\frac{\partial E_z}{\partial y}-i_y\frac{\partial E_z}{\partial x}=-j\omega\mu_o(i_xH_x+i_yH_y) \quad (1)$$

$$\nabla\times\vec{H}=i_z\left(\frac{\partial H_y}{\partial x}-\frac{\partial H_x}{\partial y}\right)=i_zj\omega\epsilon_o\epsilon E_z \quad (2)$$

$$\nabla\vec{E}=0 \quad (3)$$

$$\nabla\vec{H}=0 \quad (4)$$

and from (1), (2), and (3)

$$\nabla\times\nabla\times\vec{E}=-\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)E_z=\omega^2\mu_o\epsilon_o\epsilon E_z. \quad (5)$$

The boundary conditions are

$E_{\text{tang}}=E_z$ is continuous, $H_{\text{tang}}=H_y$ is continuous. (6)

Normalized H-field distributions, obtained with (1), are also given in Table I.

TABLE I
Normalized Field Distribution in Waveguide Containing Dielectric Slabs

Region \ Modes \ Frequency \ Field		$E_z/E_o,\ H_x/H_o=H_x\omega\mu_o/kE_o$			$jH_y/H_o=jH_y\omega\mu/kE_o$		
		$0\leq K/B<1$	$K/B=1$	$1<K/B\leq\sqrt{\epsilon}$	$0\leq K/B<1$	$K/B=1$	$1<K/B\leq\sqrt{\epsilon}$
I $0\leq\phi\leq\alpha$	all	$A\sin P\phi$	$A\phi$	$A\,\text{sh}\lvert P\rvert\phi$	$A\frac{P}{K}\cos P\phi$	$A\frac{1}{K}$	$A\frac{\lvert P\rvert}{K}\text{ch}\lvert P\rvert\phi$
II $\alpha\leq\phi\leq 1-\gamma$	all	$D\sin(Q\phi+\theta)$			$D\frac{Q}{K}\cos(Q\phi+\theta)$		
III	odd	$C\cos P(1-\phi)$	C	$C\,\text{ch}\lvert P\rvert(1-\phi)$	$C\frac{P}{K}\sin P(1-\phi)$	0	$-C\frac{\lvert P\rvert}{K}\text{sh}\lvert P\rvert(1-\phi)$
$1-\gamma\leq\phi\leq 1$	even	$C\sin P(1-\phi)$	$C(1-\phi)$	$C\,\text{sh}\lvert P\rvert(1-\phi)$	$-C\frac{P}{K}\cos P(1-\phi)$	$-C\frac{1}{K}$	$-C\frac{\lvert P\rvert}{K}\text{ch}\lvert P\rvert(1-\phi)$

The fields of Table I, together with (5), give for the propagation constants the relations

$$P^2 = B^2 - K^2 \tag{7}$$

$$Q^2 = \epsilon B^2 - K^2 \tag{8}$$

$$P^2 = Q^2 - (\epsilon - 1)B^2. \tag{9}$$

For any $Q^2>0$ there exists a $P^2 \gtrless 0$, the sign depending on $(\epsilon-1)B^2$. $Q^2>0$ describes a sinusoidal field distribution in the dielectric slab; $P^2>0$ describes a sinusoidal field distribution in the empty waveguide region. For $P^2<0$ the field distributions in the empty waveguide regions are hyperbolic functions which describe a quasi-exponential decay. $P^2=0$ gives the intermediate function where E_z and H_x have a constant slope in the empty part of the guide and where H_y is constant. In this particular case, the fields in the empty region are described by functions as ϕ exp $(P\phi)=\phi$, since $P=0$. When $Q^2<0$, $P^2<0$ also. Then the boundary conditions (6) are violated. H_y is no longer continuous. Therefore, $Q^2 \geq 0$ for all TE_{no} modes. $Q^2=0$ is reached at infinitely high frequencies. The assumptions of Table I cover only and all allowed field distributions for TE_{no} modes and are thus justified.

These field distributions are illustrated by Fig. 2 for the first odd $(n=1)$ and first even $(n=2)$ mode. The even modes $(n=2, 4, 6, \cdots)$ of the waveguide under consideration with two symmetrically placed dielectric slabs, where $2w$ is the waveguide width, correspond to the odd and even modes $(n'=n/2=1, 2, 3, \cdots)$ of a waveguide with only one dielectric slab, where w is the waveguide width; that is, the left half of Fig. 1 only. With the frequency increasing from cutoff to infinity, K/B increases from 0 to $\sqrt{\epsilon}$. For $K=B$ the field distribution between the two dielectric slabs represents a pure TEM field for all odd TE_{no} modes in the waveguide considered.

The determinantal equations for $B(K)$—e.g., cutoff frequencies for $K=0$—or $K(B)$—propagation constants for given frequencies—are found by expressing the widths of the empty sections of the guide in equivalent widths of a guide completely filled with the dielectric. The field distribution in the slabs is not changed by this replacement. The total electrical width $(2w_e)$ of this equivalent guide is $n\pi$ for a TE_{no} mode.

At the boundary between regions I and II of Fig. 1, Table I and (6) yield for frequencies $0 \leq K/B < 1$ and with $\rho_1 = P\alpha$, $\rho_2 = Q\alpha + \theta$

$$E_0 A \sin \rho_1 = E_0 D \sin \rho_2 \tag{10}$$

$$jE_0 AP \cos \rho_1/\omega\mu_0 w = jE_0 DQ \cos \rho_2/\omega\mu_0 w \tag{11}$$

$$\boldsymbol{Z}_x(\alpha) = -E_z(\alpha)/jH_y(\alpha) = j\frac{\omega\mu_0 w}{P}\operatorname{tg}\rho_1 = j\frac{\omega\mu_0 w}{Q}\operatorname{tg}\rho_2. \tag{12}$$

$\omega\mu_0 w/P = Z_p$ and $\omega\mu_0 w/Q = Z_Q$ are the transverse wave

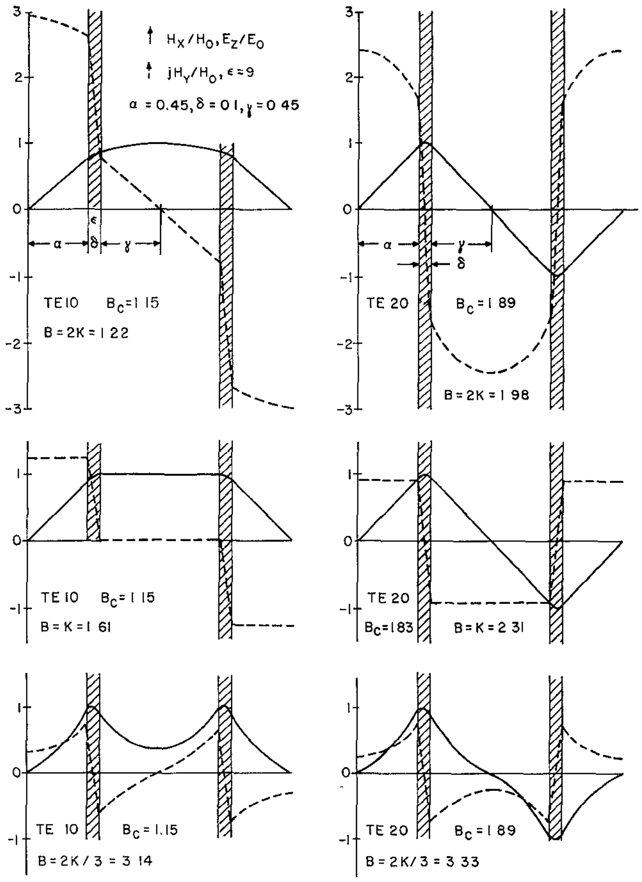

Fig. 2. Normalized field distribution in waveguide with dielectric slabs.

impedances of the empty and loaded waveguide regions, respectively, for waves traveling in $\pm x$ direction. $\boldsymbol{Z}_x(\alpha)$ is the impedance experienced by a wave traveling into the shorted waveguide of impedance Z_P and length ρ_1 or impedance Z_Q and length ρ_2. From (12) follows the equivalent length

$$\rho_2 = \operatorname{arctg}\left(\frac{Q}{P}\operatorname{tg}\rho_1\right). \tag{13}$$

Similar considerations for higher frequencies, odd and even modes, and both slab boundaries lead, for the various waveguide regions, to the actual and equivalent electrical widths of Table II. For $0 \leq K/B < 1$, ρ_5 is defined for odd modes by $1/P$ tg $\rho_4 = 1/Q$ tg ρ_5, for even modes by tg $\rho_4/P =$ tg ρ_5/Q, and for other frequencies accordingly. Table II is the skeleton for the computer program used to determine the cutoff frequencies $B(K=0)$ and propagation constants $K(B)$. The determinantal equation is in both cases

$$\rho_6 = n\pi/2 \quad \text{for} \quad TE_{no} \text{ modes}. \tag{14}$$

TABLE II
ELECTRICAL WIDTHS IN WAVEGUIDES CONTAINING DIELECTRIC SLABS

Width \ Frequency		$0 \leq K/B < 1$	$K/B = 1$	$1 < K/B \leq \sqrt{\epsilon}$
I	ρ_1	$P\alpha$	α	$\|P\|\alpha$
Equivalent I	ρ_2	$\operatorname{arctg}\left(\frac{Q}{P}\operatorname{tg}\rho_1\right)$	$\operatorname{arctg}(Q\rho_1)$	$\operatorname{arctg}\left(\frac{Q}{\|P\|}\operatorname{th}\rho_1\right)$
II	ρ_3	$Q\delta$	$Q\delta$	$Q\delta$
III	ρ_4	$P\gamma$	odd modes: 0; even modes: γ	$\|P\|\gamma$
Equivalent III	ρ_5	$\operatorname{arctg}\left[\left(\frac{Q}{P}\right)^{((-1)^n)}\operatorname{tg}\rho_4\right]$	odd modes: 0; even modes: $\operatorname{arctg}(Q\rho_4)$	$\operatorname{arctg}\left[(-1)^n\left(\frac{Q}{\|P\|}\right)^{((-1)^n)}\operatorname{th}\rho_4\right]$
I + II + III	ρ_6	$\rho_2+\rho_3+\rho_5$	$\rho_2+\rho_3+\rho_5$	$\rho_2+\rho_3+\rho_5$

TABLE III
PHASE ANGLE AND RELATIVE AMPLITUDE OF FIELDS IN WAVEGUIDES CONTAINING DIELECTRIC SLABS

Modes \ Frequency		$0 \leq K/B < 1$	$K/B = 1$	$1 < K/B \leq \sqrt{\epsilon}$
θ	all	$\rho_2 - Q\alpha$	$\rho_2 - Q\alpha$	$\rho_2 - Q\alpha$
A/D	all	$\sin\rho_2/\sin\rho_1$	$\sin\rho_2/\rho_1$	$\sin\rho_2/\operatorname{sh}\rho_1$
C/D	odd	$\sin(\rho_2+\rho_3)/\cos\rho_4$	$\sin(\rho_2+\rho_3)$	$\sin(\rho_2+\rho_3)/\operatorname{ch}\rho_4$
C/D	even	$\sin(\rho_2+\rho_3)/\sin\rho_4$	$\sin(\rho_2+\rho_3)/\rho_4$	$\sin(\rho_2+\rho_3)/\operatorname{sh}\rho_4$

Applying (6) to the boundaries of regions I/II and II/III of Fig. 1 yields the phase angle θ and the relative amplitudes of Table I. They are given in Table III in terms of the electrical widths defined in Table II.

Impedances

The power flowing through the waveguide is, with $S=2wh$ as the waveguide cross section,

$$P_y = \oint \tfrac{1}{2}(E_z \times H_x^*)dS = 2wh\frac{E_0H_0}{2}\int_0^1 g^2(\phi)d\phi \quad (15)$$

where $g(\phi)=E_z/E_0=H_x/H_0$ as given in Table I. The "wave" impedance of the guide is, with $Z_0=\sqrt{\mu_0/\epsilon_0}$,

$$Z_w = \frac{E_z}{H_x} = \frac{\omega\mu_0}{k} = Z_0\frac{B}{K}\cdot \quad (16)$$

Various definitions for "characteristic" impedances for empty waveguides have been proposed and discussed [1], [4], [18]. Applying three common definitions [1] to the dielectric loaded waveguide yields for the impedance based on the *total* longitudinal current

$$Z_i = 2P_y\Big/\left(2wH_0\int_0^1 |g(\phi)|\,d\phi\right)^2$$
$$= Z_0\frac{B}{K}\frac{h}{2w}\int_0^1 g^2(\phi)d\phi\Big/\left(\int_0^1 |g(\phi)|\,d\phi\right)^2, \quad (17)$$

for the impedance based on the *maximum* voltage

$$Z_v = (E_0h)^2/2P_y = Z_0\frac{B}{K}\frac{h}{2w}\Big/\int_0^1 g^2(\phi)d\phi, \quad (18)$$

and for the impedance based on the ratio of maximum voltage by total current

$$Z_r = \sqrt{Z_vZ_i} = Z_0\frac{B}{K}\frac{h}{2w}\Big/\int_0^1 |g(\phi)|\,d\phi. \quad (19)$$

The integrals in the impedance expressions may be written as

TABLE IV
INTEGRALS OF CHARACTERISTIC IMPEDANCE FUNCTIONS FOR TE_{10} AND TE_{20}

Int.	Modes \ Frequency	$0 \leq K/B < 1$	$K/B = 1$	$1 < K/B \leq \sqrt{\epsilon}$
Int I	all ($n = 1, 2$)	$\frac{\alpha}{2}\left[1 - \frac{\sin 2\rho_1}{2\rho_1}\right]$	$\frac{\alpha}{2}\left[\frac{2}{3}\alpha^2\right]$	$\frac{\alpha}{2}\left[\frac{\text{sh}\,\rho_1\,\text{ch}\,\rho_1}{\rho_1} - 1\right]$
Int II	all ($n = 1, 2$)	$\frac{\delta}{2}\left[1 - \frac{\sin\left[2(\rho_2 + \rho_3)\right] - \sin 2\rho_2}{2\rho_3}\right]$		
Int III	odd ($n = 1$)	$\frac{\gamma}{2}\left[1 + \frac{\sin 2\rho_4}{2\rho_4}\right]$	$\frac{\gamma}{2}\left[2\right]$	$\frac{\gamma}{2}\left[\frac{\text{sh}\,\rho_4\,\text{ch}\,\rho_4}{\rho_4} + 1\right]$
	even ($n = 2$)	$\frac{\gamma}{2}\left[1 - \frac{\sin 2\rho_4}{2\rho_4}\right]$	$\frac{\gamma}{2}\left[\frac{2}{3}\gamma^2\right]$	$\frac{\gamma}{2}\left[\frac{\text{sh}\,\rho_4\,\text{ch}\,\rho_4}{\rho_4} - 1\right]$
In I	all ($n = 1, 2$)	$\alpha(1 - \cos\rho_1)/\rho_1$	$\alpha(\alpha/2)$	$\alpha(\text{ch}\,\rho_1 - 1)/\rho_1$
In II	all ($n = 1, 2$)	$\delta(\cos\rho_2 - \cos(\rho_2 + \rho_3))/\rho_3$		
In III	odd ($n = 1$)	$\gamma\,(\sin\rho_4)/\rho_4$	γ	$\gamma\,(\text{sh}\,\rho_4)/\rho_4$
	even ($n = 2$)	$\gamma\,(1 - \cos\rho_4)/\rho_4$	$\gamma\,(\gamma/2)$	$\gamma\,(\text{ch}\,\rho_4 - 1)/\rho_4$

$$\int_0^1 g^2(\phi)d\phi = A^2 \,\text{Int I} + D^2 \,\text{Int II} + C^2 \,\text{Int III} \quad (20)$$

$$\int_0^1 \left| g(\phi) \right| d\phi = A \,\text{In I} + D \,\text{In II} + C \,\text{In III}. \quad (21)$$

For higher-order modes the direction of the longitudinal current flow changes over the guide width. The absolute value $|g(\phi)|$ in (17), (19), and (21) takes this reversal into account. The evaluation of $\int |g(\phi)| d\phi$ is generally quite complicated. For the TE_{10} and TE_{20} modes, however, $|g(\phi)| = g(\phi)$, and Int I, Int II, Int III and In I, In II, In III are as given in Table IV.

For TE_{10} and TE_{20} modes the normalizing field strength E_0 is the maximum field strength in the guide, when

$$C = 1 \quad \text{if } \rho_2 + \rho_3 < \pi/2$$
$$D = 1 \quad \text{if } \rho_2 + \rho_3 \geq \pi/2. \quad (22)$$

The asymptotic values for the characteristic impedances are easily obtained for these two modes with

$$\lim_{B \to \infty} \left(\frac{K}{B}\right) = \sqrt{\epsilon} \quad \text{for all } \epsilon \text{ and } \delta \quad (23)$$

$$\lim_{\epsilon \to 1} \int_0^1 g^2 d\phi = \tfrac{1}{2}, \quad \lim_{\epsilon \to 1} \int_0^1 g d\phi = \frac{2}{\pi} \quad \text{for all } \delta \text{ and } B \quad (24)$$

$$\lim_{\delta \to 0} \int_0^1 g^2 d\phi = \tfrac{1}{2}, \quad \lim_{\delta \to 0} \int_0^1 g d\phi = \frac{2}{\pi} \quad \text{for all } \epsilon \text{ and } B \quad (25)$$

$$\lim_{\delta \to 1} \int_0^1 g^2 d\phi = \tfrac{1}{2}, \quad \lim_{\delta \to 1} \int_0^1 g d\phi = \frac{2}{\pi} \quad \text{for all } \epsilon \text{ and } B \quad (26)$$

$$\lim_{B \to \infty} \int_0^1 g^2 d\phi = \frac{\delta}{2}, \quad \lim_{B \to \infty} \int_0^1 g d\phi = \frac{2\delta}{\pi} \quad \text{for } \delta > 0 \text{ and } \epsilon > 1. \quad (27)$$

For $\epsilon = 1$, $\delta = 0$, $\delta = 1$ one obtains the expressions for the empty or homogeneously filled waveguide

$$Z_i = \frac{1}{2}\left(\frac{\pi}{2}\right)^2 Z_0 \frac{B}{K} \frac{h}{2w}, \quad Z_v = 2Z_0 \frac{B}{K} \frac{h}{2w},$$

$$P_y = \frac{2wh}{4} E_0 H_0. \quad (28)$$

For $B \to \infty$ and $\delta > 0$, $\epsilon > 1$ one obtains

$$Z_i = \frac{1}{2}\left(\frac{\pi}{2}\right)^2 Z_0 \frac{1}{\sqrt{\epsilon}} \frac{h}{2d}, \quad Z_v = 2Z_0 \frac{1}{\sqrt{\epsilon}} \frac{h}{2d},$$

$$P_y = \frac{2dh}{4} E_0 H_0 \tag{29}$$

where $2dh$ is the cross section of the dielectric slabs. At relatively high frequencies, all the power flow is concentrated in the dielectric slabs.

Computer Results

Normalized cutoff frequencies have been calculated for TE_{n0} modes of the guide of Fig. 1 with $n = 1, 2, 3, 4, 6$ for six relative dielectric constants (2.25, 4, 9, 12.25, 16, 25), fifteen slab thicknesses (including 0% and 100% filling factor), and a maximum of eleven positions of the slab in the guide. Normalized propagation constants have been calculated for TE_{10} and TE_{20} modes between their respective cutoff frequency and a frequency somewhat above the second- and fourth-order mode cutoff frequency of the empty guide, respectively—again for the relative dielectric constants given above, five slab thicknesses (5, 10, 15, 25, 40% filling factor), and a maximum of eight slab positions. The position parameter $\alpha + \delta/2$ gives the distance between the left guidewall and the center plane of the left slab as a fraction of half the guide width. For the odd modes the position is varied between the slabs touching the guidewalls ($\alpha + \delta/2 = \delta/2$) and the slabs touching each other in the center of the guide ($\alpha + \delta/2 = 1 - \delta/2$). For even modes it suffices, because of the symmetry of the field distribution, to vary the position between the slabs touching the wall and moving them half way toward each other ($\alpha + \delta/2 = 0.5$). For even modes the cutoff frequencies and propagation constants are the same for $\alpha + \delta/2 = \tau$ and $\alpha + \delta/2 = 1 - \tau$.

The results for the full range of parameters are given numerically and graphically in a General Electric publication [16]. In this paper, selected results are presented graphically. Normalized cutoff frequencies are shown in Fig. 3. Normalized propagation constants are shown in Fig. 4.

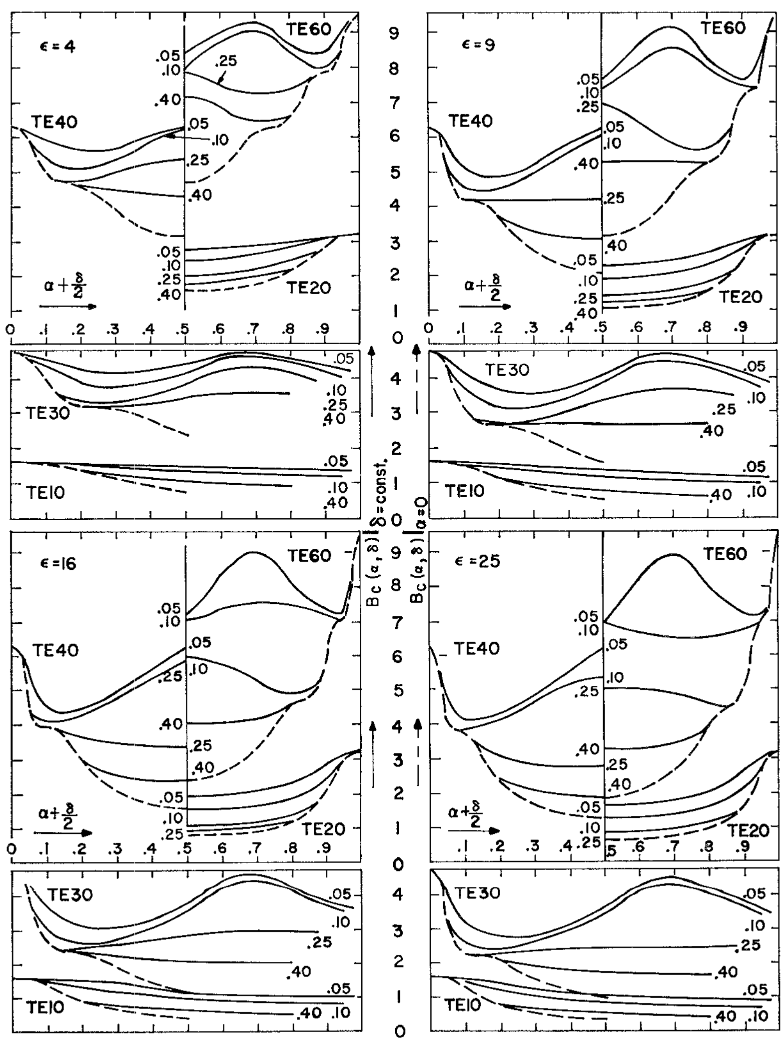

Fig. 3. Normalized cutoff frequencies with normalized slab thickness (δ) as parameter.

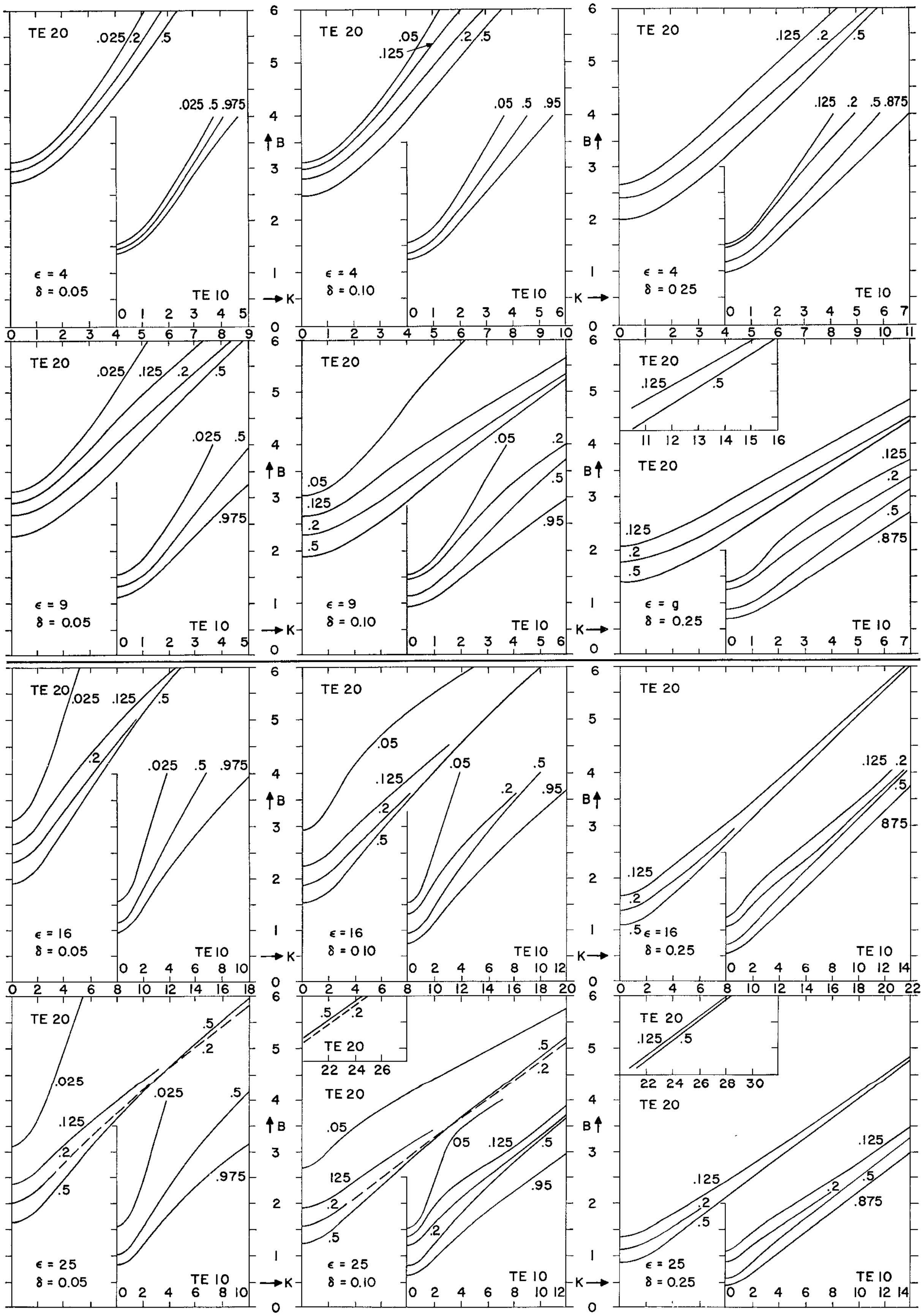

Fig. 4. Normalized propagation characteristics with normalized slab position $(\alpha+\delta/2)$ as parameters.

Examples

a) Guide WR 137, width 1.372 inches, two slabs each 0.069 inch thick, 0.137 inch between left wall and center of left slab, relative dielectric constant $\epsilon=9$, $w=\text{width}/2=0.686$ inch, $\delta=0.1$, $\alpha+\delta/2=0.2$. *Wanted:* TE_{10} and TE_{20} cutoff frequencies and guide wavelengths at 5.46 GHz.

One finds $B_c(TE_{10})\approx 1.44$, $B_c(TE_{20})\approx 2.31$. With $\lambda_c=2\pi w/B_c$ one obtains $\lambda_c(TE_{10})\approx 7.6$ cm, $\lambda_c(TE_{20})\approx 4.74$ cm, and $f_c(TE_{10})\approx 3.95$ GHz, $f_c(TE_{20})\approx 6.2$ GHz. At 5.46 GHz one finds $B=2$ and $K(TE_{10})\approx 1.58$, corresponding to $\lambda_g=2\pi w/K\approx 6.94$ cm; no propagation for the TE_{20} mode.

b) Guide WR 90, width 0.9 inch, one slab with $\epsilon=16$, 0.090 inch thick, 0.45 inch between wall and center of slab. *Wanted:* cutoff frequencies of the two lowest-order modes and guide wavelength at 10 GHz for the lowest-order mode. $w'=\text{width}=0.9$ inch, $\delta=0.10$, $\alpha+\delta/2=0.5$. $B_c(TE_{10}')=B_c(TE_{20})$ of guide with two slabs and width 1.8 inches and $w=\text{width}/2$.

One finds $B_c(TE_{10}')\approx 1.51$, $B_c(TE_{20}')=B_c(TE_{40})\approx 5.82$. With $\lambda_c=2\pi w'/B_c$ one obtains $\lambda_c(TE_{10}')\approx 9.52$ cm, $\lambda_c(TE_{20}')\approx 2.46$ cm, and $f_c(TE_{10}')\approx 3.16$ GHz, $f_c(TE_{20}')\approx 12.45$ GHz. At 10 GHz one finds $B=2\pi w'/\lambda_0\approx 4.8$ and $K\approx 13.03$, yielding $\lambda_g\approx 1.1$ cm.

c) The TE_{20} solutions with $\alpha+\delta/2=0.5$ are equivalent to the TE_{10} solutions with the two slabs touching each other, where $\alpha+\delta/2=1-\delta/2$, i.e., $2K(TE_{10},\ B,\ \alpha+\delta/2=1-\delta/2)=K(TE_{20},\ 2B,\ \alpha+\delta/2=0.5)$, $2B_c(TE_{10},\ \alpha+\delta/2=1-\delta/2)=B_c(TE_{20},\ \alpha+\delta/2=0.5)$.

Discussion of Results

The influence of a thin slab on the cutoff frequencies (Fig. 3) becomes stronger the closer the slab is placed to the lines of maximum electric field strength in the empty guide. With increasing slab thickness this influence is less and less related to the empty guide field distribution. For thick slabs it is weakest when the slabs touch the guidewalls; for odd modes it is strongest when the slabs are in the center of the guide and touch each other, and for even modes it is strongest when each slab is in the center of half a guide width. At a certain thickness the cutoff frequency is nearly independent of the slab position (see $TE_{30,40,60}$).

The effect of the dielectric slab on the field distribution in the waveguide is to concentrate within the slab with increasing frequency an increasing fraction of the total energy flowing through the guide. The phase velocity approaches asymptotically $1/\sqrt{\epsilon}$ times the velocity of light in free space. The parametric dependence of the E-field distribution is illustrated by Fig. 5. An interesting effect occurs when propagation characteristics cross over each other, as seen in Fig. 4 for $\epsilon=25$, $\delta=0.10$. If the empty regions in half the waveguide (I and III, α and γ in Fig. 1) are not equal, a wave may treat them at high frequencies as essentially equal with the value of the narrower region. This waveguide shows then a larger propagation constant and field concentration at a given frequency than a guide with $\alpha=\gamma$ does. An E-field distribution for such a case is shown in the lowest part of Fig. 5.

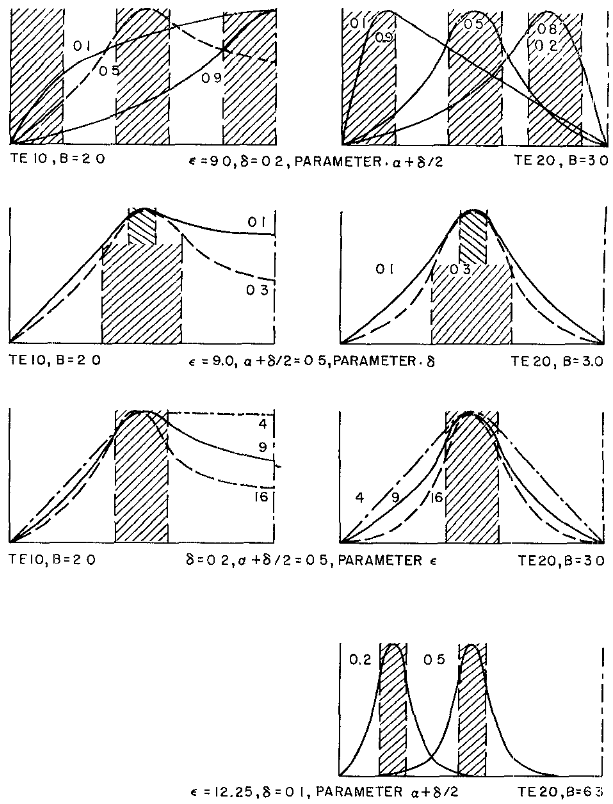

Fig. 5. Normalized E-field distribution in waveguide with dielectric slabs.

In the case of thin slabs, where the cutoff frequencies for higher modes are strongly influenced by the empty guide field distribution, the propagation characteristics for a given ϵ and δ may all cross in one point. At that frequency the propagation constant is independent of the slab position (see [16], TE_{40}, $\epsilon=9$, $\delta=0.2$). Employing lossy dielectrics one may use the frequency-dependent field concentration to construct dissipative field displacement filters [17].

Examples of the parametric dependence of normalized characteristic impedances

$$z_i=Z_i\Big/\left(Z_0\,\frac{h}{2w}\right)\quad\text{and}\quad z_v=Z_v\Big/\left(Z_0\,\frac{h}{2w}\right)$$

are given in Fig. 6. Examples of the parametric dependence of the fractional bandwidths, i.e., the ratios of cutoff frequencies

$$\Omega_{21}=\frac{B_c(TE_{20})}{B_c(TE_{10})}\quad\text{and}\quad\Omega_{42}=\frac{B_c(TE_{40})}{B_c(TE_{20})}$$

are shown in Fig. 7.

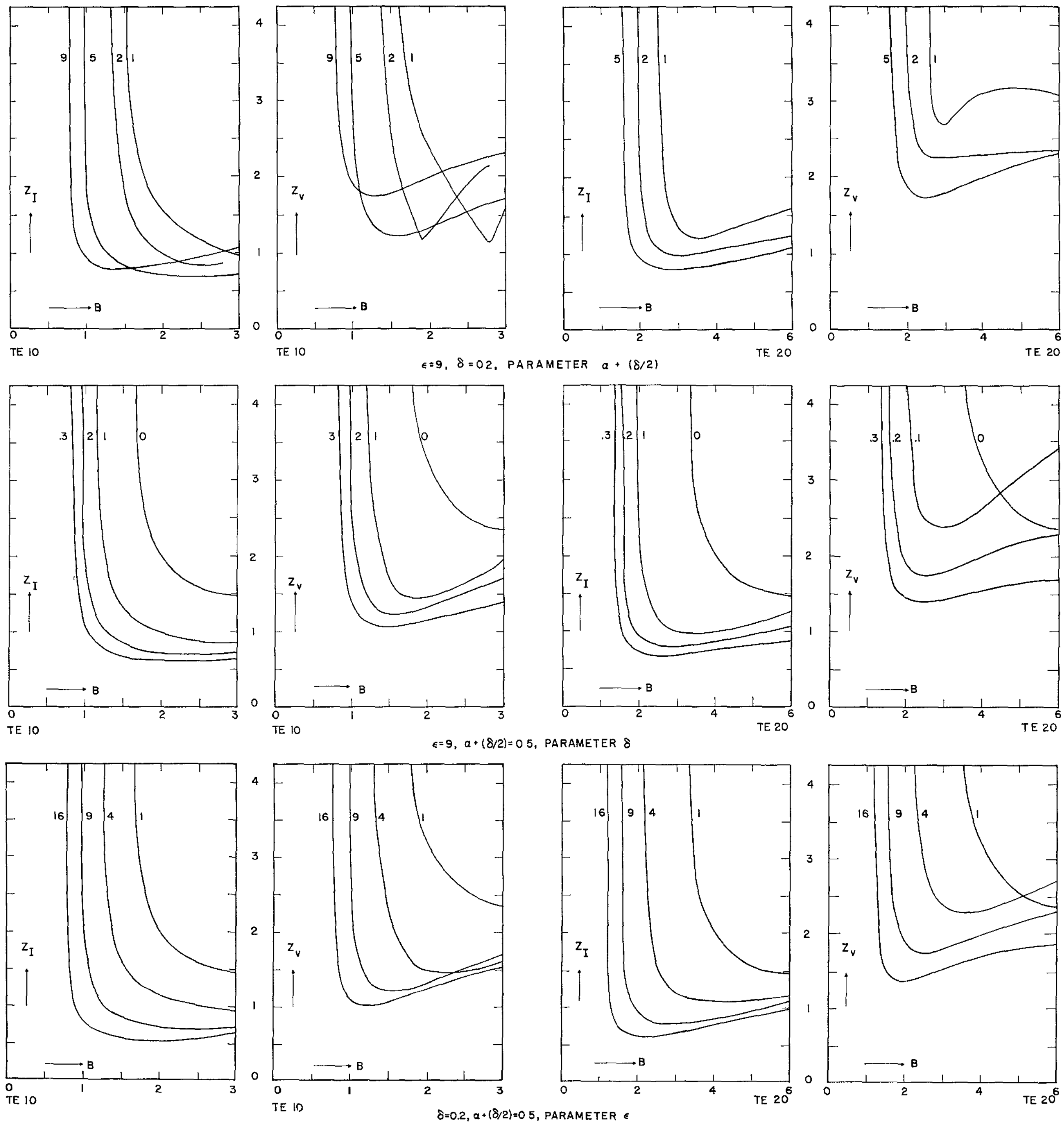

Fig. 6. Normalized characteristic impedances of waveguide with dielectric slabs.

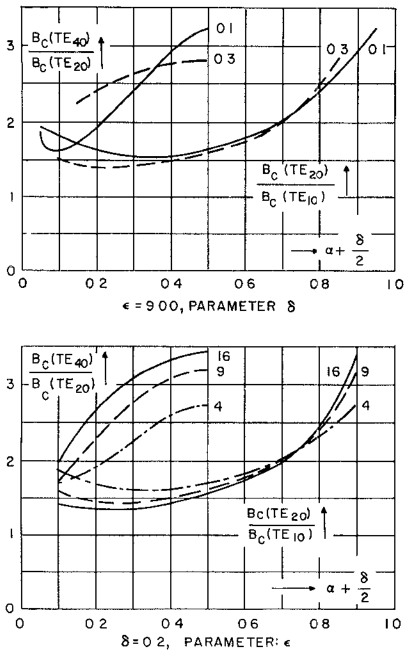

Fig. 7. Fractional bandwidths of waveguide with dielectric slabs.

The ratio of the magnetic field components, the ellipticity,

$$\text{ELL}(\phi) = H_x(\phi)/jH_y(\phi)$$

at the slab boundaries I/II and II/III approaches unity asymptotically, as illustrated in Fig. 8 for TE_{10} and TE_{20} modes for various parameters.

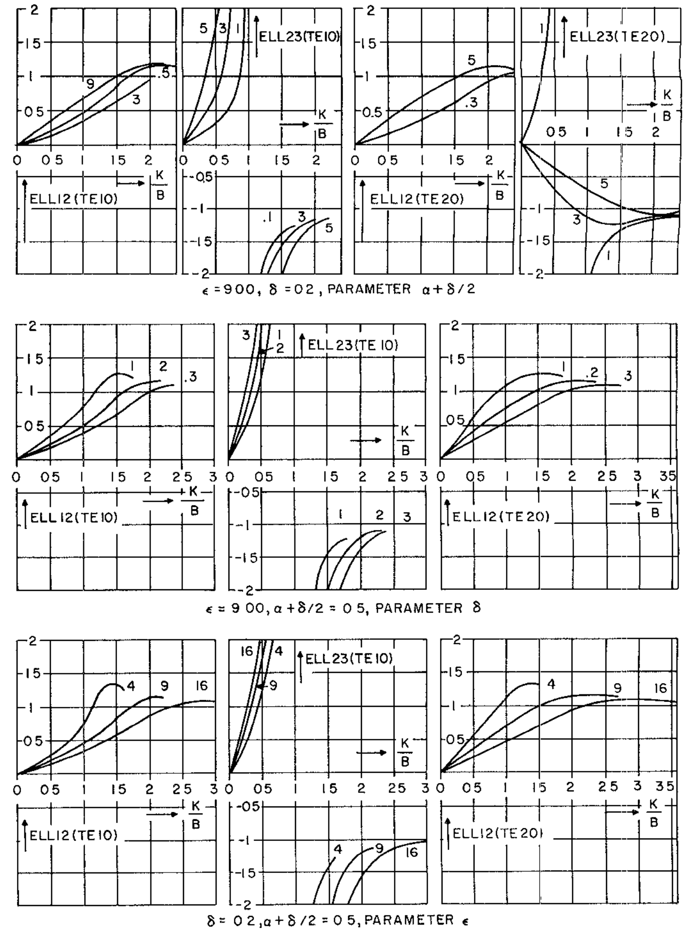

Fig. 8. Ellipticity at slab boundaries in waveguide with dielectric slabs.

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